High-Q Frequency Stable Dual-Mode Whispering Gallery Sapphire Resonator

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***Abstract* — The design and experimental test of a dual mode high-Q Whispering Gallery (WG) sapphire resonator is presented. Dual mode operation is realized by designing the $WGE_{7,0,0}$ and the $WGH_{9,0,0}$ mode near 9 GHz and separated in frequency by approximately 80 MHz. Design was achieved by implementing Finite Element software, which is shown to agree very well with measurement. Due to the anisotropy of sapphire, WGH and WGE modes have different Temperature Coefficient of Frequency (TCF). We show that the difference frequency can be used to stabilize the temperature, resulting in a temperature limited frequency stability of better than one part in 10^{13}. This type of resonator has the potential to improve substantially the close to the carrier phase noise in current state-of-the-art low noise oscillators.**

I. INTRODUCTION

To create an oscillator with exceptional frequency stability, the resonator must have immunity to temperature fluctuations, as well as a high quality factor. The high quality-factor of Whispering Gallery (WG) sapphire resonators (>10^5 at room temperature) has enabled the lowest phase noise [1] oscillators in the microwave regime at Fourier frequencies near 1 kHz. However, below 1 Hz Fourier frequency an oscillator based on a Sapphire loaded Cavity (SLC) has an inferior performance to a quartz oscillator[2, 3]. The Temperature Coefficient of Permittivity (TCP) is anisotropic for sapphire, and is 50 ppm/K for Transverse Electric (TE) modes and 70 ppm/K for Transverse Magnetic (TM) modes. This mechanism allows temperature fluctuations to transform to resonator frequency fluctuations. In this paper we investigate the possibility of utilizing the difference in temperature dependence to stabilize the temperature of the resonator and reduce temperature induced frequency fluctuations. This can be achieved by designing a WGE (quasi TE) and a WGH (quasi TM) mode close in frequency and using the frequency difference to control the temperature.

The idea of having two electromagnetic oscillators sharing the same resonator (dual mode) and locked to its different modes in order to improve immunity to ambient temperature fluctuations, has been applied previously to quartz crystal oscillators[4]. In this work we investigate how a similar technique may be applied to high-Q WG modes in a SLC resonator-oscillator. First we present the design of the resonator, which was implemented with the aid of a separation of variables technique [5, 6], and later studied in more rigorously with finite element design[7]. Then the frequency and Q-factor dependence is compared to finite element calculations, which shows such a resonator may be designed accurately using this technique. Finally we show that the performance of such a resonator can improve substantially the close to the carrier frequency noise induced by temperature fluctuations.

II. SLC DESIGN

The design of the SLC, along with the magnetic field density plot of the $WGE_{7,0,0}$ mode is shown in fig. 1. The sapphire cylinder is housed in a copper cavity of 60 mm diameter and 40 mm height. The sapphire itself is 39.4 mm by 20 mm, with a 4.3 mm hole through the center. Two built in copper posts, attached to the top and bottom of the copper cavity support the cavity. The top and bottom posts hold the sapphire firmly by squashing indium between the top and bottom post. This also aids the thermal contact of the sapphire to the cavity.

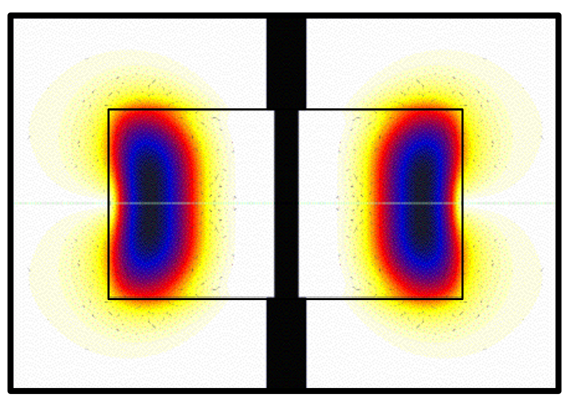

Fig. 1. Scale design of the Dual-Mode WG sapphire resonator. The sapphire cylinder is housed in a copper cavity of 60 mm diameter and 40 mm height. The colored field pattern represents the magnetic field density of the

$WGE_{7,0,0}$ mode inside the cavity as calculated by finite element analysis.

To model the frequency-temperature dependence the anisotropic permittivity and expansion coefficients of sapphire need to be known as a function of temperature [8]. Using these values, a program was written, that automatically generated a 2D finite element mesh as a function of temperature according to the calculated size and anisotropic complex permitivities. Calculations of the frequency and Q-factor were performed at intervals of 10 K between 250 to 300 K. Due to the symmetry of a cylindrical resonator, only 1/4 of the resonator structure was needed in the finite element mesh.

II. Experimental Results

The resonator is housed in a vacuum chamber thermally contacted to a thermoelectric Peltier module. One side of Peltier element is contacted to the cavity, while the other is heat sunk to the outside. The temperature is controlled via a servo, which includes a Lock-In Amplifier, PID controller and a thermistor heat sensor in an AC bridge. The error signal is fed back to the Peltier element, which controls the temperature to the set point determined by a variable resistor in the AC bridge. When the set point was varied the resonator was allowed to come to equilibrium, this time depended on the temperature but was typically of order 1 hour. Once the resonator came to equilibrium, the frequency, Q-factor and temperature was measured. The temperature was measured via a calibrated thermistor connected to the opposite side of the cavity to the Peltier and sensing thermistor.

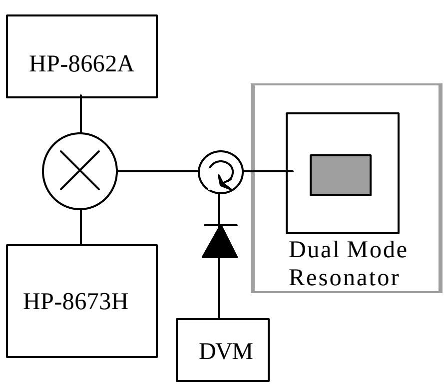

Fig. 2. Measurement setup for the dual mode resonator.

The $WGE_{7,0,0}$ mode was excited by coupling to the radial electric field (E_r) with an electric field probe half way between the ends of the cavity through a hole on the cylindrical cavity wall. The $WGH_{9,0,0}$ mode was excited by coupling to the axial electric field (E_z) from the top of the cavity. The resonator was analyzed in reflection with values of coupling of order 0.1. A stable swept frequency was created using an 8673H HP synthesizer (2-12.4 GHz) mixed with an 8662A HP synthesizer (10 kHz - 1.2 GHz) as shown in fig. 2. To determine which side band was being measured a microwave frequency counter was used to read out the frequency. An automatic program controlled the sweep and fitted a Lorentzian to the shape of the reflected signal. The resonant frequencies were determined by the frequency of minimum reflection, and the loaded Q-factor was determined by the Lorentzian fit. Once the resonator was in equilibrium, the program was left to repetitively measure the frequency, Q-factor and coupling. Fig. 3 and 4 compare measurement with calculated values. Scatter is shown in the Q-factor measurements from the repetitive measurements, and is a good representation in the uncertainty of the measurement due to the uncertainty in the Lorentzian fit.

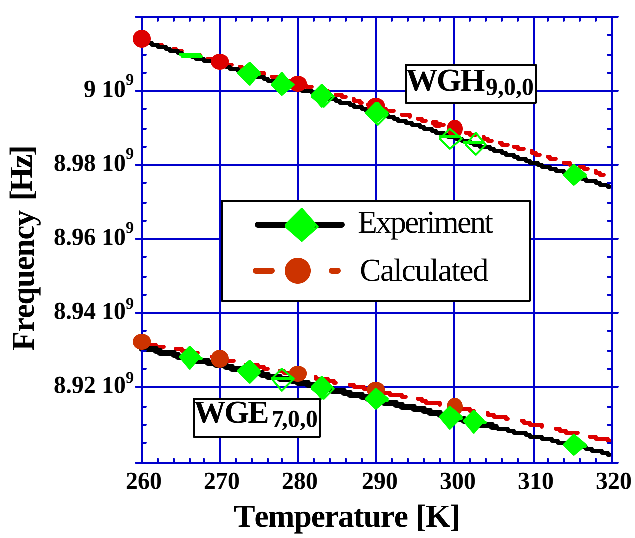

Fig. 3. Comparison of the measured and calculated frequency of the $WGE_{7,0,0}$ and $WGH_{9,0,0}$ modes.

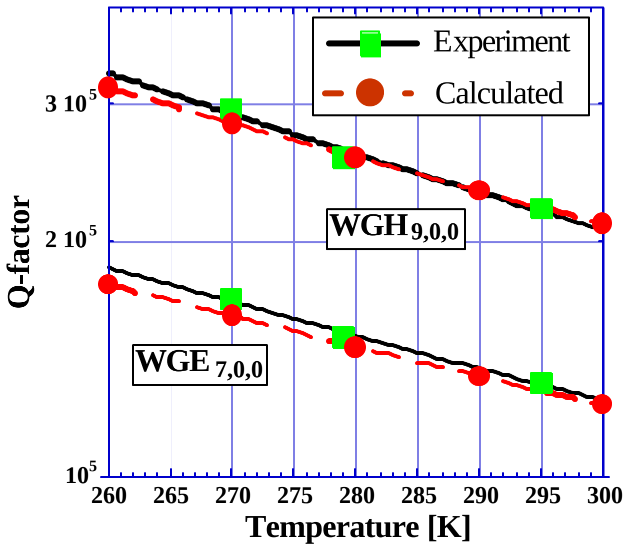

Fig. 4. Comparison of the measured and calculated unloaded Q-factors of the $WGE_{7,0,0}$ and $WGH_{9,0,0}$ modes.

III. FREQUENCY STABILITY

In this section we calculate the frequency stability (or Allan deviation) of the oscillator utilizing the Dual Mode resonator. First, we characterize the relation between the rms temperature and rms frequency fluctuations of the dual modes by,

$$\delta f_{WGE,H} = \left|\frac{df}{dT}\right|_{WGE,H} \delta T_{SLC} \qquad (1).$$

Here, δT_{SLC} is the temperature fluctuations of the SLC, $df/dT|_{WGE,H}$ is the TCF and $\delta f_{WGE,H}$ is the frequency fluctuation of the resonator induced by temperature. For example, at 280 K we measured $df/dT|_{WGE}$ = -450 kHz/K and $df/dT|_{WGE}$ = -650 kHz/K (calculated from data in fig. 3).

To calculate the potential frequency stability a feed back control system is assumed to make the beat between the WGE and WGH mode equal to an external reference oscillator. In this case the frequency fluctuations become entirely determined by the frequency fluctuations of the reference oscillator, δf_{REF}. The relation between the temperature fluctuations of the SLC and the reference oscillator becomes,

$$\delta T_{SLC} = \frac{\delta f_{REF}}{\left| \left.{}^{df}/_{dT}\right|_{WGH} - \left.{}^{df}/_{dT}\right|_{WGE} \right|} \qquad (2).$$

Substituting (2) into (1) gives;

$$\delta f_{WGE,H} = \left| \frac{\left.\frac{df}{dT}\right|_{WGE,H}}{\left.{}^{df}/_{dT}\right|_{WGH} - \left.{}^{df}/_{dT}\right|_{WGE}} \right| \delta f_{REF} \qquad (3)$$

As follows from (3), frequency fluctuations of a microwave signal and those of reference oscillator are linearly related. For this reason, averaging both types of frequency fluctuations over measurement time, τ, results in a linear relationship between frequency stability of microwave and reference signals, given by;

$$\sigma_{y_{WGE,H}}(\tau) = \Re_{WGE,H}\, \sigma_{y_{REF}}(\tau) \qquad (4).$$

Where, $\sigma_{y_{WGE,H}}(\tau)$ is the Allan standard deviations of a microwave oscillator operating at frequency of the WGE or WGH mode, $\sigma_{y_{REF}}(\tau)$ is the Allan standard deviation of the reference oscillator, and $\Re_{WGE,H}$ is given by;

$$\Re_{WGE,H} = \left| \frac{(f_{WGE} - f_{WGH})\left.\frac{df}{dT}\right|_{WGE,H}}{\left.{}^{df}/_{dT}\right|_{WGH} - \left.{}^{df}/_{dT}\right|_{WGE}} \right| \frac{1}{f_{WGE,H}} \qquad (5).$$

The factor $\Re_{WGE,H}$ is the frequency noise reduction factor of the dual mode resonator assuming that $f_{REF}=|f_{WGE}-f_{WGH}|$. Note that this value is only dependent on the frequencies and TCF's of the dual mode resonator, and in-contrast to atomic clocks, the frequency stability of a microwave signal produced by a dual mode oscillator can be much better that that of a flywheel reference source.

The separation of mode frequencies at 280 K was measured to be 79 MHz, with the frequency of the $WGE_{7,0,0}$ mode equal to 8.921 GHz and the frequency of the $WGH_{9,0,0}$ mode equal to 9.000 GHz. Thus, from (4) $\Re_{WGE}$ was calculated to be 0.032 and $\hat{A}_{WGH}$ was calculated to be 0.023. This means that the frequency stability of the reference oscillator needs to be of order 5×10^{-12} to obtain a frequency stability of order 10^{-13}, which is feasible with current quartz technology. The best quartz oscillators at 80 MHz exhibit a frequency instability of order 5×10^{-13} [3], which means in principle a frequency stability of order 10^{-14} could even be possible using this technique.

IV. CONCLUSION

Accurate design of a Dual Mode WG SLC resonator has been achieved using finite element analysis. The $WGE_{7,0,0}$ and $WGH_{9,0,0}$ modes were designed to operate near 9 GHz, with a frequency separation accurately designed to be near

80 MHz. The difference in the TCF's of the modes may be used to temperature stabilize the resonator resulting in a high frequency stability. This technique may be used as an alternative technique to annulling the TCF with dielectric or paramagnetic techniques, without degrading the Q-factor of the resonator.

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